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Five-dimensional Trinification Improved

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Abstract

We present improved models of trinification in five dimensions. Unified symmetry is broken by a combination of orbifold projections and a boundary Higgs sector. The latter can be decoupled from the theory, realizing a Higgsless limit in which the scale of exotic massive gauge fields is set by the compactification radius. Electroweak Higgs doublets are identified with the fifth components of gauge fields and Yukawa interactions arise via Wilson loops. The result is a simple low-energy effective theory that is consistent with the constraints from proton decay and gauge unification.

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I. INTRODUCTION

Extra dimensions provide a variety of new tools for building realistic Grand Unified Theories (GUTs). In orbifold compactifications, for example, different components of a GUT multiplet may be assigned different parities under reflections about the orbifold fixed points. Judicious choices can yield a particle spectrum in which all unwanted states (for example, color-triplet Higgs fields) appear at or near the compactification scale $1/R$. A related technique that has received some attention in the context of electroweak symmetry breaking is the Higgsless mechanism [1, 2]. In this approach, a more general set of boundary conditions are employed, allowing for the reduction in the rank of the gauge group. These boundary conditions can be thought of as arising from a boundary Higgs sector that has been decoupled from the theory. Interestingly, in this decoupling limit, the spectrum of massive gauge fields is determined by $1/R$ rather than the boundary vacuum expectation values (vevs) [3]. While electroweak symmetry breaking clearly necessitates the reduction in rank of the gauge group, the same is true of GUTs with rank greater than four. This was the motivation for the study of boundary breaking in trinified theories [4], one of the simplest unified theories of rank six. Other recent work on trinified theories in extra dimensions appears in Ref. [5].

While Ref. [4] explored the usefulness of generalized boundary conditions in breaking a simple unified theory of rank greater than four, the models presented there had a number of shortcomings: electroweak symmetry breaking was still accomplished by introducing chiral Higgs multiplets and a fine-tuning was required to keep these fields in the low-energy spectrum. In this letter, we present simpler models that avoid these problems. Electroweak Higgs doublets will be identified as components of gauge fields, an economical approach known as gauge-Higgs unification in the literature [6, 7], and these Higgs fields will remain light down to the weak scale due to an R-symmetry [8]. In addition, we present one construction in which an additional gauge group factor provides both for a unified boundary condition on the standard model gauge couplings and also serves as an origin for the electroweak Higgs fields. This yields a trinified theory without the cumbersome (though entirely conventional) cyclic symmetry whose only purpose is to maintain the equality of GUT-scale gauge couplings. The two models we present are consistent with the constraints from proton decay

and gauge coupling unification.

II. $SU(3)^3 \ltimes \mathbf{Z}_3$

Conventional trinification is based on the gauge group $G_T = SU(3)_C \times SU(3)_L \times SU(3)_R \ltimes Z_3$. The discrete symmetry cyclically permutes the group labels C,L, and R, which maintains a single gauge coupling g at the unification scale. Gauge and matter fields transform under the **24**- and **27**-dimensional representations, respectively, with decompositions

$$\begin{aligned} \mathbf{24} &= (\mathbf{8},\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1},\mathbf{8}) \ , \\ \mathbf{27} &= (\mathbf{1},\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{1},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}},\mathbf{1}) \ , \end{aligned} \tag{2.1}$$

under the C, L, R gauge factors. In the usual Gell-Mann basis, weak SU(2) is generated by T_L^a for $a = 1 \ldots 3$, while hypercharge, in its standard model normalization, is generated by

$$Y = -\frac{1}{\sqrt{3}}(T_L^8 + \sqrt{3}T_R^3 + T_R^8) \ . \tag{2.2}$$

With the hypercharge gauge coupling identified as $\sqrt{3/5}g$, the choices above yield the standard GUT-scale prediction $\sin^2\theta_W = 3/8$. This is phenomenologically acceptable in the present context, given the new boundary corrections to unification [3] that we expect generically in extra-dimensional models.

We first consider a model in five dimensions (5D) with G_T chosen as the bulk gauge symmetry. We compactify the extra dimension on an $S^1/(Z_2 \times Z_2')$ orbifold, labelled by the coordinate y. Defining $y' \equiv y + \pi R/2$, points related by the translation $y \to y + 2\pi R$ and by the reflections $y \to -y$ and $y' \to -y'$, are identified. The physical region in y is thus reduced to the interval $[0, \pi R/2]$. In addition, we assume $\mathcal{N} = 1$ supersymmetry in 5D. Bulk gauge fields thus form $\mathcal{N} = 2$ 4D hypermultiplets consisting of $\mathcal{N} = 1$ vector $V(A^\mu, \lambda)$ and chiral $\Phi(\sigma + iA_5, \lambda')$ multiplets at each Kaluza-Klein (KK) level. All matter fields are placed on the $\pi R/2$ brane for simplicity.

We now show that the electroweak Higgs doublets of the minimal supersymmetric standard model (MSSM) can be identified with some of the A_5 components of the gauge multiplets. Under the two orbifold parities, we assume the bulk fields transform as follows:

$$\begin{aligned} V(x^\mu, -y) &= P\, V(x^\mu, y)\, P^{-1} \, , \qquad V(x^\mu, -y') = P'\, V(x^\mu, y')\, P'^{-1} \\ \Phi(x^\mu, -y) &= -P\, \Phi(x^\mu, y)\, P^{-1} \, , \quad \Phi(x^\mu, -y') = -P'\, \Phi(x^\mu, y')\, P'^{-1} \ . \end{aligned} \tag{2.3}$$

Here P and P' are 3×3 matrices that act in gauge group space and have eigenvalues of ± 1. Noting that the supersymmetric bulk action requires the terms $\mathcal{S}_{5D} \supset \int d^4\theta \frac{2}{g^2} Tr(\sqrt{2}\partial_5 + \Phi^\dagger)e^{-V}(-\sqrt{2}\partial_5 + \Phi)e^V$ [9], one sees that $\partial_5 V$ and Φ should have the same transformation properties under the orbifold parities. Therefore, although components within a gauge multiplet can transform differently under the parity operations, the relative sign of the vector and chiral multiplets is uniquely determined. With the notation $(P, P') = (P_C \oplus P_L \oplus P_R, P'_C \oplus P'_L \oplus P'_R)$ we choose

$$\begin{aligned} P_C &= diag(1,1,1), \quad P_L = diag(1,1,-1), \quad P_R = diag(1,1,-1), \\ P'_C &= diag(1,1,1), \quad P'_L = diag(1,1,-1), \quad P'_R = diag(1,1,1). \end{aligned} \tag{2.4}$$

Parity assignments for the component fields immediately follow:

$$V_C: \left(\begin{array}{cc|c} (+,+) & (+,+) & (+,+) \\ (+,+) & (+,+) & (+,+) \\ \hline (+,+) & (+,+) & (+,+) \end{array}\right), \qquad \Phi_C: \left(\begin{array}{cc|c} (-,-) & (-,-) & (-,-) \\ (-,-) & (-,-) & (-,-) \\ \hline (-,-) & (-,-) & (-,-) \end{array}\right), \tag{2.5}$$

$$V_L: \left(\begin{array}{cc|c} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ \hline (-,-) & (-,-) & (+,+) \end{array}\right), \qquad \Phi_L: \left(\begin{array}{cc|c} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ \hline (+,+) & (+,+) & (-,-) \end{array}\right), \tag{2.6}$$

$$V_R: \left(\begin{array}{cc|c} (+,+) & (+,+) & (-,+) \\ (+,+) & (+,+) & (-,+) \\ \hline (-,+) & (-,+) & (+,+) \end{array}\right), \qquad \Phi_R: \left(\begin{array}{cc|c} (-,-) & (-,-) & (+,-) \\ (-,-) & (-,-) & (+,-) \\ \hline (+,-) & (+,-) & (-,-) \end{array}\right). \tag{2.7}$$

As we will see shortly, fields that are odd under P have vanishing wave functions at $y = 0$, while those that are odd under P' vanish at $y = \pi R/2$. It follows that the gauge symmetry that is operative at the $\pi R/2$ fixed point is $SU(3)_C \times SU(2)_L \times U(1)_L \times SU(3)_R$, a fact that we will use later. Only fields that are even under both P and P' have massless zero modes, from which we conclude that the total effect of the orbifold projection is to reduce the bulk gauge symmetry to $SU(3)_C \times SU(2)_L \times U(1)_L \times SU(2)_R \times U(1)_R$. Crucially, two $\mathrm{SU}(2)_L$ doublets in the chiral multiplet Φ_L retain massless zero modes, and it follows immediately from Eq. (2.2) that these have hypercharges $Y = \pm\frac{1}{2}$. We identify these superfields with the MSSM Higgs doublets.

We break the remaining gauge symmetry down to that of the MSSM using generalized boundary conditions. To illustrate this approach consider a gauge field A^μ that is even under reflections about $y = 0$. This implies that the 5D wave function for the k^{th} mode has the form

$$A_\mu(x^\nu, y) \sim \cos(M_k\, y) A_\mu^{(k)}(x^\nu) \ , \tag{2.8}$$

for y in the interval $0 \leq y \leq \pi R/2$. Imposing the boundary condition

$$\partial_5 A^\mu(y = \pi R/2) = V A^\mu(y = \pi R/2)\,, \tag{2.9}$$

one obtains the following transcendental equation for M_k

$$M_k \tan(M_k \pi R/2) = -V \ . \tag{2.10}$$

In the large V limit the KK spectrum is well approximated by

$$M_k \approx M_c \frac{(2k+1)}{2}(1 + \frac{M_c}{\pi V} + \cdots), \quad k = 0, 1, \ldots\,, \tag{2.11}$$

where we define the compactification scale $M_c \equiv 2/R$. Thus, in the limit $V \rightarrow \infty$, the spectrum reduces to a tower whose low lying states are $M_c/2$, $3M_c/2$, $5M_c/2$, *etc.* This is shifted by $M_c/2$ relative to the tower one would obtain if V were set to zero. The symmetry breaking parameter V has dimensions of mass and can be associated with products of the form $g^2 v^2$, where g is a five-dimensional gauge coupling and v a boundary vev. Since v generically sets the scale of the physical states in the boundary symmetry-breaking sector, the limit $V \rightarrow \infty$ corresponds to the decoupling of the boundary Higgs fields from the theory. It is worth noting that in the supersymmetric case, the spectrum of the additional scalar and fermionic components of Φ and V are the same as in Eq. (2.11), as a consequence of gauge invariance and unbroken supersymmetry [3].

In the present context, we could introduce two **27** boundary Higgs fields, whose $(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$ components have appropriate vevs to break SU(3)^3 down to the standard model gauge group. However, we have already noted that the orbifold projection has reduced the gauge symmetry to $SU(3)_C \times SU(2)_L \times U(1)_L \times SU(3)_R$ at the $\pi R/2$ brane. This allows us to choose much simpler Higgs representations at this boundary to implement the symmetry breaking:

$$\chi_1 = \bar{\chi}_1 = \begin{pmatrix} 0 \\ 0 \\ v_1 \end{pmatrix}, \quad \chi_2 = \bar{\chi}_2 = \begin{pmatrix} 0 \\ v_2 \\ v_3 \end{pmatrix} . \tag{2.12}$$

Here the field χ is an $\mathrm{SU}(3)_R$ triplet with $\mathrm{U}(1)_L$ charge $+1/\sqrt{3}$, and $\bar{\chi}$ has conjugate quantum numbers. Both χ_i and $\bar{\chi}_i$ are singlets under color and $SU(2)_L$, and are together anomaly free. They represent the one relevant row of the $(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$ representation used to break the unified symmetry in conventional trinified models. The real vevs shown in χ_1 and χ_2 are completely general choices, while we assume the same vevs for the barred fields. (The latter choice is consistent with D-flatness.) These vevs are sufficient to break the remaining gauge symmetry down to that of the standard model. Note, however, that a realistic potential may require additional fields.

How do these boundary vevs affect the spectra of fields transforming as $(+,+)$, $(-,-)$, $(-,+)$, and $(+,-)$ under our orbifold reflections? First, the fields $A^\mu(x^\nu, y)$ whose wave functions are odd at the $y = \pi R/2$ brane (corresponding to parities $(\pm,-)$), vanish at that endpoint. Therefore, these KK towers are unaffected by the boundary vevs, and are given by $M_n(-,-) = (n+1)M_c$ and $M_n(+,-) = (n+1/2)M_c$, for $n = 0, 1, \ldots$. As discussed above, the $(+,+)$ fields acquire a massive tower $M_n(+,+,V \to \infty) = (n+1/2)M_c$. Finally, consider the $(-,+)$ fields. Since these wave functions are odd at the $y = 0$ brane they have the general form

$$A_\mu(x^\nu, y) \sim \sin(M_k\, y) A_\mu^{(k)}(x^\nu) \ . \tag{2.13}$$

Imposing the boundary condition in Eq. (2.9) yields the transcendental equation

$$M_k \cot(M_k \pi R/2) = V \ , \tag{2.14}$$

which implies a KK tower

$$M_k \approx M_c\, k\, (1 + \frac{M_c}{\pi V} + \cdots)\,, \quad k = 1, 2, \ldots\,, \tag{2.15}$$

in the large V limit.

In the present case, the boundary conditions following from the existence of the χ and $\bar{\chi}$ vacuum expectation values may be written

$$\partial_5 A_\mu^i(x^\nu, \pi R/2) = V_{ij} A_\mu^j(x^\nu, \pi R/2) \ , \tag{2.16}$$

where V_{ij} is a matrix in the space of the $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ gauge fields. The entries of this matrix were considered explicitly in Ref. [4], and have the form $\sum_i c_i g^2 v_i^2$, where the v_i are defined in Eq. (2.12) and the c_i are numerical coefficients. The precise form of V_{ij} and the

values of the c_i are irrelevant for the present analysis since we will always take the $g^2 v_i^2$ to be large compared to M_c. In this limit, the spectra of KK modes become independent of these details, and we obtain one of six possible towers already discussed: $(+,-)$, $(-,-)$, $(+,+,V=0)$, $(+,+,V\to\infty)$, $(-,+,V=0)$, and $(-,+,V\to\infty)$.

As noted earlier, matter fields are located at the $\pi R/2$ fixed point. Although the SU(3)^3 **27**s decompose into a direct sum of representations under the unbroken gauge symmetry at $\pi R/2$, all of these components must be retained in order to have an anomaly-free theory that reproduces complete MSSM generations in the low-energy theory. We now show that the exotic matter content of the **27**s become massive via couplings to the boundary Higgs fields χ and $\bar{\chi}$, and decouple from the theory as the vevs v_i are taken large. We first decompose the **27** under the unbroken SU(3)$_C\times$SU(2)$_L\times$U(1)$_L\times$ SU(3)$_R$ symmetry at $y=\pi R/2$,

$$\mathbf{27} = L(\mathbf{1},\mathbf{2},\bar{\mathbf{3}})_q + e^c(\mathbf{1},\mathbf{1},\bar{\mathbf{3}})_{-2q} + q^c(\bar{\mathbf{3}},\mathbf{1},\mathbf{3})_0 + Q(\mathbf{3},\mathbf{2},\mathbf{1})_{-q} + B(\mathbf{3},\mathbf{1},\mathbf{1})_{2q} \;\;, \qquad (2.17)$$

where the U(1)$_L$ charge $q=1/(2\sqrt{3})$. The notation serves as a reminder of the embedding of standard model fields. In addition, the L multiplet contains a vector-like pair of exotic lepton doublets, (E^0, E^-) and (E^+, E^{0c}), e^c contains a pair of exotic singlets, N^c and N, while q^c contains the right-handed partners of the exotic left-handed, charge $-1/3$ quarks that make up the multiplet B in its entirety. GUT scale mass terms arise via the superpotential couplings

$$W = h_L(L^i_\alpha L^j_\beta \bar{\chi}_\gamma \epsilon^{\alpha\beta\gamma}\epsilon_{ij}) + \frac{1}{2} h_Q (q^{c\alpha}_{\;\;i} B^j \bar{\chi}_\beta \delta^i_j \delta^\beta_\alpha)\,. \qquad (2.18)$$

Expanding Eq.(2.18) produces the low-energy matter content of minimal trinified theories. The right-handed d and B quarks mix leaving one linear combination massless to be identified with the physical d_R quark. Similarly, only one linear combination of the left-handed lepton doublets receives a mass from the first term in Eq. (2.18). Thus, the low-energy spectrum consists of the particle content of the MSSM, as well as the singlets N and N^c. These can be made massive as well by including higher-dimension operators in the superpotential of the form, $(e^c\chi_i)^2/\Lambda$, where Λ is the cutoff of the effective theory. Thus, unlike the model in Ref. [4], no additional fields need to be included at the boundary to rid the low-energy theory of the singlets.

Having recovered the MSSM particle content, we now consider how to obtain Yukawa couplings involving the electroweak Higgs doublets. Since we have identified the Higgs dou-

blets with components of the bulk adjoint chiral superfield Φ, which transforms nonlinearly under a gauge transformation (*i.e.*. $\Phi \to e^{\Lambda}(\Phi - \sqrt{2}\partial_5)e^{-\Lambda}$.), no local Yukawa couplings are possible. However, the solution to this problem is well known in the literature on 5D gauge-Higgs unification models: one may couple the Higgs doublets to the matter fields at the fixed point via Wilson loop operators [7, 8]. The Wilson line operator $\mathcal{H} = \mathcal{P}exp(\int_{y_i}^{y_f} \frac{1}{\sqrt{2}}\Phi dy)$, where $\mathcal{P}$ represents the path ordered product, is a nonlocal object that transforms linearly under the 5D gauge transformation at points y_i and y_f, $\mathcal{H} \to e^{\Lambda}|_{y_f}\mathcal{H}e^{-\Lambda}|_{y_i}$. Choosing $y_i = y_f = \pi R/2$ and a path that wraps around the extra dimension, one obtains a Wilson loop operator that transforms linearly at the orbifold fixed point where our matter fields are located: $\mathcal{H} \to e^{\Lambda}\mathcal{H}e^{-\Lambda}$, with $\Lambda \equiv \Lambda(x^\mu, y = \pi R/2)$. We focus on the doublet components of $\mathcal{H}$, which we call $H(\mathbf{1},\mathbf{2},\mathbf{1})_{3q}$ and $\bar{H}(\mathbf{1},\mathbf{2},\mathbf{1})_{-3q}$, using the notation of Eq. (2.17). Yukawa couplings originate at the $\pi R/2$ brane via the interactions

$$W = \frac{1}{\Lambda}\bar{\chi}Le^cH + \frac{1}{\Lambda}\bar{\chi}Qq^cH + \frac{1}{\Lambda^2}\chi_1\chi_2Qq^c\bar{H} \ , \tag{2.19}$$

after the χ and $\overline{\chi}$ fields develop vevs. Here Λ is a cutoff of the effective theory. Note that in the decoupling limit $v_i \to \Lambda \to \infty$, none of the terms in Eq. (2.19) are suppressed; this is an indication that the low-energy theory is restricted only by standard model gauge symmetry at $y = \pi R/2$.

We resolve the μ problem in our model by using the $U(1)_R$ symmetry of the bulk action. Under this symmetry, the superspace coordinate θ transforms with charge $+1$, while V and Φ are neutral. An $H\bar{H}$ term is not allowed since the superpotential must have R-charge -2. We may induce a small μ parameter by coupling the Higgs fields to a singlet X with R-charge -2, via the superpotential coupling $XH\bar{H}$. The μ parameter is generated if the X field develops a vev, which can happen naturally due to supersymmetry-breaking effects, as in the next-to-minimal supersymmetric standard model. Note that this mechanism works assuming we impose only a discrete subgroup of $\mathrm{U}(1)_R$, which avoids any unwanted R-axions. Assuming the χ and $\bar{\chi}$ have R-charge zero and each matter field -1, then the Yukawa couplings in Eq. (2.19) are allowed and a Z_4 subgroup of $\mathrm{U}(1)_R$ is sufficient.

Finally, we consider the issue of gauge coupling unification. The possible towers of KK modes are described by either $(n + 1/2)M_c > 0$ or $nM_c > 0$, for n an integer. The supersymmetric beta functions for the fields charged under the standard model gauge groups are

	(b_3, b_2, b_1)	$(\tilde{b}_3, \tilde{b}_2, \tilde{b}_1)$
$(V,\Phi)_{321}$	(-9,-6,0)	(-6,-4,0)
$(V,\Phi)_{(1,2,\pm 1/2)}$	(0,1,$\frac{3}{5}$)	(0,-2,-$\frac{6}{5}$)
$(V,\Phi)_{(1,1,\pm 1)}$	-	(0,0,$-\frac{24}{5}$)
Matter	(6,6,6)	-
Total	(-3,1,$\frac{33}{5}$)	(-6,-6,-6)

TABLE I: Contributions to the beta function coefficients from the zero modes (b_i) and the KK levels ($\tilde{b}_i$). Here the Φ represent chiral multiplets in the adjoint representation. Results in the second and third lines represent sums over all fields with the stated quantum numbers.

shown in Table I. Note that only two exotic (V,Φ) multiplets, with charges $(\mathbf{1},\mathbf{1},\mathbf{1})$ and $(\mathbf{1},\mathbf{1},-\mathbf{1})$, respectively, under SU(3)$_C\times$ SU(2)$_W\times$U(1)$_Y$ have KK towers that are shifted down by $M_c/2$ due to the boundary Higgs vevs. Notice that if all the KK towers were aligned, they would contribute universally to the gauge running. As first pointed out in Ref. [3], the shifted spectra contribute as a tower of threshold corrections; while the power-law running is still universal, the running of the differences $\alpha_i^{-1}-\alpha_j^{-1}$ is logarithmic. With the notation $\delta_i(\mu)=\alpha_i^{-1}(\mu)-\alpha_1^{-1}(\mu)$, for $i=2$ or 3, the differential running above the first KK threshold is given by

$$\delta_i(\mu)=\delta_i(M_c/2)-\frac{1}{2\pi}R_i(\mu)\ , \tag{2.20}$$

where

$$R_2(\mu)=-\frac{28}{5}\log(\frac{\mu}{M_c/2})-\frac{12}{5}\sum_{0<nM_c<\mu}\log(\frac{\mu}{nM_c})+\frac{12}{5}\sum_{0<(n+1/2)M_c<\mu}\log(\frac{\mu}{[n+1/2]M_c}), \tag{2.21}$$

$$R_3(\mu)=-\frac{48}{5}\log(\frac{\mu}{M_c/2})-\frac{12}{5}\sum_{0<nM_c<\mu}\log(\frac{\mu}{nM_c})+\frac{12}{5}\sum_{0<(n+1/2)M_c<\mu}\log(\frac{\mu}{[n+1/2]M_c})\,. \tag{2.22}$$

Note that the last two terms in each equation above would cancel if the KK-towers were aligned, and one would obtain the differential running of the MSSM. Numerical study of these equations reveal that unification is preserved, but that the scale of unification M_U is delayed. For example, for $M_C=4\times 10^{14}$ GeV we find $M_U\approx 8\times 10^{16}$ GeV, which is approximately the 5D Planck scale. For $M_C=2\times 10^{16}$ GeV we find $M_U=2.8\times 10^{16}$ GeV,

which simply demonstrates that there is a limit in which most of the KK towers do not contribute and MSSM unification is recovered. For 4×10^{14} GeV$< M_U < 2 \times 10^{16}$ GeV we find that the α^{-1} unify at well below the 1% level, ignoring possible boundary effects. Thus, our extra-dimensional construction does not lead to any problems with successful gauge unification. Discussion of other possible corrections to unification may be found in Ref. [4] and will not be discussed further here.

Finally, we note that there is no proton decay in this model. In ordinary trinification, proton decay is mediated by colored Higgses that are part of a **27**. In our model, the smaller gauge symmetry at the $\pi R/2$ fixed point allowed us to include symmetry breaking fields in much smaller representations, without dangerous colored components. Since there is no proton decay from the gauge sector of trinified theories, our model is safe from these effects.

III. $\mathbf{SU(9) \times SU(3)^3}$

Before concluding, we wish briefly to present an alternative starting point that can provide a common origin for the GUT-scale equality of gauge couplings (without the Z_3 symmetry) and the existence of the electroweak Higgs doublets. We consider an $SU(9) \times SU(3)^3$ bulk gauge theory on a $S^1/(Z_2 \times Z_2')$ orbifold. The SU(3)$_C\times$SU(3)$_L\times$ SU(3)$_R$ symmetry of our previous model is identified with the diagonal subgroup of an SU(3)^3 living within SU(9) and the other SU(3)^3 factor, so that

$$\frac{1}{g^2_{(C,L,R)}} = \frac{1}{g^2_{SU(9)}} + \frac{1}{g^2_{(C',L',R')}} \, . \tag{3.1}$$

Here C',L', and R' refer to the three SU(3) factors present before symmetry breaking. If these SU(3) gauge groups are somewhat strongly coupled, then Eq. (3.1) leads to an approximate unified boundary condition for the diagonal subgroup. This is precisely the idea of "unification without unification" described in Ref. [10]. Note that the bulk gauge symmetry can be thought of as a two-site deconstructed sixth dimension, with symmetry broken at a boundary. Generalizations to replicated SU(9) factors are also interesting, since the primed gauge couplings do not have to be made particularly large. In any case, the

SU(9) vector multiplet decomposes under the diagonal $\mathrm{SU}(3)^3$ subgroup as

$$V_9:\left(\begin{array}{c|c|c}(\mathbf{8},\mathbf{1},\mathbf{1}) & (\mathbf{3},\bar{\mathbf{3}},\mathbf{1}) & (\mathbf{3},\mathbf{1},\bar{\mathbf{3}}) \\ \hline (\bar{\mathbf{3}},\mathbf{3},\mathbf{1}) & (\mathbf{1},\mathbf{8},\mathbf{1}) & (\mathbf{1},\mathbf{3},\bar{\mathbf{3}}) \\ \hline (\bar{\mathbf{3}},\mathbf{1},\mathbf{3}) & (\mathbf{1},\bar{\mathbf{3}},\mathbf{3}) & (\mathbf{1},\mathbf{1},\mathbf{8})\end{array}\right) . \tag{3.2}$$

We know from ordinary trinification that fields with the quantum numbers of Higgs doublets live in the $(\mathbf{1},\mathbf{3},\bar{\mathbf{3}})$ representation and its conjugate. We therefore wish to find parity assignments that preserve these elements of the chiral adjoint Φ as well. With parity transformations defined as in Eq. (2.3), we choose

$$\begin{aligned} P_{SU(9)} = diag(1,1,1,1,1,1,1,-1,1), \qquad & P'_{SU(9)} = diag(1,1,1,-1,-1,1,1,1,1) \ , \\ P_C = diag(1,1,1),\ \ P_L \ = \ diag(1,1,-1),\ \ & P_R = diag(1,1,-1), \\ P'_C = diag(1,1,1),\ \ P'_L \ = \ diag(1,1,1),\ \ \ & P'_R = diag(1,1,1). \end{aligned} \tag{3.3}$$

One finds, for example, that the $(\mathbf{1},\mathbf{3},\bar{\mathbf{3}})$ components of the SU(9) chiral adjoint Φ_9 has parities

$$\Phi_9(\mathbf{1},\mathbf{3},\bar{\mathbf{3}}):\left(\begin{array}{ccc}(-,+) & (+,+) & (-,+) \\ (-,+) & (+,+) & (-,+) \\ (-,-) & (+,-) & (-,-)\end{array}\right) , \tag{3.4}$$

which indicates the location of one of the Higgs doublets. Aside from the corresponding $(+,+)$ entries in the $(\mathbf{1},\bar{\mathbf{3}},\mathbf{3})$ block, all other components of Φ_9 have no zero modes.

The orbifold parities in Eq. (3.3) break the SU(9) symmetry to $SU(8)\times U(1)$ at the $y=0$ fixed point, $SU(7)\times SU(2)\times U(1)$ at $y=\pi R/2$ and to $SU(6)\times SU(2)\times U(1)\times U(1)'$ overall. The $SU(3)^3$ factors are broken to $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ overall, but are unbroken at $y=\pi R/2$. Thus, the most natural way to include matter fields is by introducing complete $\mathbf{27}$'s at the $\pi R/2$ fixed point.

The breaking of the remaining gauge symmetry down to that of the MSSM can be done with a boundary Higgs sector, as in our previous model. To determine the necessary representations, we may pretend the $\mathrm{SU}(3)^3$ factor is embedded in another SU(9), and use the fact that a Higgs $\Sigma\sim(\mathbf{9},\bar{\mathbf{9}})$ with diagonal vevs will leave a diagonal SU(9) unbroken. A straightforward decomposition of Σ in terms of the actual gauge symmetry at $\pi R/2$, $\mathrm{SU}(7)\times\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{SU}(3)^3$, gives the desired representations. These break the remaining symmetry down to the diagonal subgroup $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_L\times\mathrm{SU}(2)'_R\times\mathrm{U}(1)'_R$.

We may recover the standard model gauge group by including SU(9) singlet, $(\mathbf{1},\mathbf{3},\bar{\mathbf{3}})$ and $(\mathbf{1},\bar{\mathbf{3}},\mathbf{3})$ boundary Higgs fields, with the same pattern of vevs found in conventional trinified theories. Yukawa couplings can arise via higher dimension operators involving the boundary Higgs fields, and are unsuppressed in the Higgsless limit, as shown in the previous model; the decoupling of exotic matter fields also works in the same way. Color-triplet components of Φ_9 exist, so that proton decay is not absent, but doublet-triplet spliting is explained naturally via the orbifold projection.

IV. CONCLUSIONS

We have presented improved models of 5D trinification. In the first model, unified symmetry was broken by a combination of orbifold projections and a boundary Higgs sector that could be decoupled from the theory. Electroweak Higgs fields appeared economically as the fifth components of gauge fields. The model demonstrated the existence of a consistent low-energy theory in which no chiral Higgs fields needed to be added to the theory in an ad hoc way. This model is free of proton decay and consistent with gauge unification. In the second model, we showed that an additional SU(9) gauge factor could provide a common origin for the unified boundary condition on the standard model gauge couplings, and the origin of the electroweak Higgs, via gauge-Higgs unification. Both models provide new and explicit realizations of 5D trinified GUTs, and demonstrate a Higgsless approach that can be applied to other unified theories with rank greater than four.

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